1.2 Fractions, Decimals, Percent

- · Prime + composite numbers
- · Divisibility Tests
- " Factors
- · Prime factorization
- · Multiples, Common multiples
- . Least Common Muitiple
- . Equivalent fractions (CD)
- . Reduce fractions
- . Round decimals
- . Convert fraction to decimal
- . Convert decimal to fraction
- · Convert percent to decimal
- · Convert decimal to percent
- · Convert percent to fraction
- . Convert fraction to percent

1.3 Number Systems + Real Number Line

- · classify natural, whole, integer, rational, and irrational numbers
- · plot points with braction or decimal parts on a number line
- · use inequalities to compare two
 numbers
- · Evaluate expressions with absolute values

2 handowts

- 1,2 examples
- 1.3 examples

Included in Scanned notes but not photoeopied Math Terminology

Explanations of all methods and solutions for all examples in this handout can be found on the class website. Use your own paper when working these problems in class.

1. Identify prime and composite numbers

Examples 1 & 2: For each of these numbers, identify whether it is prime or composite. 2, 3, 4, 5, 6, 7, 13, 23, 27

2. Divisibility tests

Examples 3, 4, 5, 6, and 7: For each of these numbers, identify whether it is divisible by 2, divisible by 3, divisible by 5, and divisible by 10.

275, 351, 390, 1236, 3589, 24579, 24686, 24687, 3568000

3. Find factors of a number

Example 8: List all the factors of 12.

4. Factor a number into a product of prime numbers OR Find the prime factorization of a number Examples 9 and 10: Find the prime factorization of 56

5. Find multiples of a number

Example 11: List the first ten multiples of the number 9.

6. Find a Common Multiple of two numbers

Example 12: Find the first two common multiples of 5 and 8

7. Find the Least Common Multiple (LCM) of two or more numbers

Example 13 and 15: Find the LCM of 15 and 20.

Example 14 and 16: Find the LCM of 27, 12, and 30.

8. Write equivalent fractions (higher terms) = Write with common denominator

Example 17: Write $\frac{2}{9}$, $\frac{5}{12}$ as equivalent fractions with a common denominator.

9. Write equivalent fractions (lower terms) = Reduce to lowest terms.

Example 18: Reduce $\frac{24}{42}$

10. Round decimals

Example 19: Round 1.69843 to the nearest thousandth.

Example 20: Round 1.69843 to the nearest hundredth.

11. Convert a fraction to a decimal

Example 21: Convert $\frac{3}{8}$ to a decimal.

Example 22: Convert $\frac{7}{9}$ to a decimal.

12. Convert a decimal to a fraction

Example 23: Write 0.13 as a fraction.

Example 24: Write 0.168 as a fraction.

13. Convert a percent to a decimal

Example 25: Write 95% as a decimal.

Example 26: Write 5.2% as a decimal.

Example 27: Write 135% as a decimal.

Example 28: Write 1.02% as a decimal.

14. Convert a decimal to a percent

Example 29: Write 0.63 as a percent.

Example 30: Write 0.006 as a percent.

Example 31: Write 72 as a percent.

Example 32: Write 1.03 as a percent.

15. Convert percent to fraction

Example 33: Write 95% as a fraction.

Example 34: Write 5.2% as a fraction.

Example 35: Write 135% as a fraction.

Example 36: Write 1.02% as a fraction.

16. Convert fraction to percent

Example 37: Write $\frac{3}{4}$ as percent.

Example 38: Write $\frac{2}{9}$ as percent.

1. Identify prime and composite numbers

A prime number is a whole number which is divisible evenly only by 1 and itself and is not 1.

A composite number is not prime, so it can be divided evenly by some number other than 1 and itself

Example 1: 2, 3, 5, 7 and 23 are prime.

Example 2: 4, 6, 18, and 27 are composite

2. Divisibility tests

A number is divisible by 2 if it is even, that is, it ends in 0, 2, 4, 6, or 8 in the ones place

A number is divisible by 5 if it ends is 0 or 5 in the ones place

A number is divisible by 10 if it ends in 0 in the ones place

A number is divisible by 3 if the sum of its digits is divisible by 3.

For each of these numbers, identify whether it is divisible by 2, divisible by 3, divisible by 5, and divisible by 10: 275, 351, 390, 1236, 3589, 24579, 24686, 24687, 3568000

Example 3:

390, 1236, 24686 and 3568000 are divisible by 2 because the digits in the ones place are even numbers. 275, 351, 3589, 24579, 24687 is not divisible by 2 because the digits in the ones place are odd numbers.

Example 4:

275, 390, and 3568000 are divisible by 5 because the digit in the ones place is either 0 or 5.

351, 1236, 3589, 24579, 24686, and 24687 are not divisible by 5.

Example 5:

390 and 3568000 are divisible by 10 because the digits in the ones place are zeros.

275, 351, 1236, 3589, 24579, 24686, 24687 are not divisible by 10 because the digits in the ones place are not zeros.

Example 6:

3+5+1=9 is divisible by 3, so 351 is divisible by 3.

3+9+0=12 is divisible by 3, so 390 is divisible by 3.

1+2+3+6=12 is divisible by 3, so 1236 is divisible by 3.

2+4+5+7+9=27 is divisible by 3, so 24579 is divisible by 3.

2+4+6+8+7=27 is divisible by 3, so 24687 is divisible by 3.

Example 7:

2+7+5=14 is not divisible by 3, so 275 is not divisible by 3.

3+5+8+9=25 is not divisible by 3, so 3589 is not divisible by 3.

2+4+6+8+6=26 is not divisible by 3, so 24686 is not divisible by 3.

3+5+6+8+0+0+0=22 is not divisible by 3, so 3568000 is not divisible by 3.

3. Find factors of a number

A factor is a number which divides into the given number.

Example 8: List all the factors of 12.

The factors of 12 are all the numbers which divide evenly into 12.

Answer: 1, 2, 3, 4, 6, 12

Solutions and Explanations to go with Practice problems handout from class.

9x9 = 81

9x10 = 90

Math 45 1.2 Review of Fractions, Decimals, and Percent, page 2

4. Factor a number into a product of prime numbers OR Find the prime factorization of a number

The prime factorization is a list of prime numbers, which, when multiplied together, equal the given number. Create a factor tree, then write the product of primes.

Caution: Don't stop factoring until all factors are prime.

Caution: Instructions may ask for the answer to be written with exponents.

Method 1: Find any two numbers which multiply to the given number. Repeat for each factor, until the last factors are prime. Caution: Don't stop dividing until all factors are prime.

Example 9: Find the prime factorization of 56

Method 2: Test potential primes in order: Does it divide by 2? (divide by 2 until it will not anymore) Then, does it divide by 3? (divide by 3 until it will not), etc. Divide until the last result is prime.

Example 10: Find the prime factorization of 56

Answer;
$$56 = \frac{3}{2} \cdot 7$$

5. Find multiples of a number

Take the number and multiply by 1, 2, 3, 4, ...

Example 11: List the first 10 multiples of 9.

Answer: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90

6. Find a Common Multiple of two numbers

A common multiple is a number which is in the list of multiples of the first number and also in the list of multiples of the second number. It's "common" to, or shared by both lists.

A common multiple is divisible by both numbers.

Example 12: Find the first two common multiples of 5 and 8

Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, ...

Multiples off 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104, ...

Answer: The first two common multiples of 5 and 8 are 40 and 80.

7. Find the Least Common Multiple (LCM) of two or more numbers

The Least Common Multiple (LCM) is the lowest number that is a common multiple of both numbers. The LCM is used to find a common denominator for adding or subtracting fractions.

<u>Method 1 (for small numbers)</u>: Choose the largest number, then check its multiples to see if they are divisible by the other given numbers. *Caution*: This method can take a long time, and you must get all the arithmetic correct.

Example 13: Find the LCM of 15 and 20.

20 is the larger number.

Multiples of 20: 20, 40, 60, 80, 100, 120, ...

Test 20: Does 15 divide into 20? No

Test 40: Does 15 divide into 40? No.

Test 60: Does 15 divide into 60? Yes.

Answer: 60 is the LCM (15, 20).

Example 14: Find the LCM of 27, 12, and 30.

30 is the largest number.

Multiples of 30: 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 330, 360, 390, 420, ...

Test 30: Does 27 divide into 30? No. (Doesn't matter that 12 doesn't, either.)

Test 60: Does 27 divide into 60? No. (Doesn't matter that 12 does.)

Test 90: Does 27 divide into 90? No. (Doesn't matter that 12 doesn't, either.)

Test 120: Does 27 divide into 120? No. (Doesn't matter that 12 does).

Test 150: Does 27 divide into 150? No. (Doesn't matter that 12 doesn't, either).

Test 180: Does 27 divide into 180? No. (Doesn't matter that 12 does).

Test 210: Does 27 divide into 210? No. (Doesn't matter that 12 doesn't, either).

Test 240: Does 27 divide into 240? No. (Doesn't matter that 12 does).

Test 270: Does 27 divide into 210? Yes. But 12 doesn't!

Keep going. Test 300, 330, 360, 390, 420, 450, 480, 510.

Answer: Trust me, you have to get to 540 before both 27 and 12 both work.

Method 2 (for large numbers): Find the prime factorizations of each number. Build the LCM from the prime factors so that each number in the list is part of the LCM. This means including the largest number of factors that appears in any one number, but no duplications from other numbers.

Exampe 15: Find the LCM of 15 and 20.

Find prime factorizations:

15 = 3.5 (We'll need one 3 and one 5)

 $20 = 2 \cdot 2 \cdot 5$ (We'll need two 2s. We already have one 5.)

Answer: LCM = $2 \cdot 2 \cdot 3 \cdot 5 = 60$.

Example 16; Find the LCM of 27, 12, and 30.

Find prime factorizations:

 $27 = 3^3$ (We'll need three 3s.)

 $12 = 2 \cdot 2 \cdot 3$ (We'll need two 2s and one 3. We already have enough 3s.)

 $30 = 2 \cdot 3 \cdot 5$ (We'll need one 2, one 3 and one 5. We have the 2 and 3 already.)

Answer: LCM = $3 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 5 = 540$.

8. Write equivalent fractions (higher terms) = Write with common denominator

Equivalent fractions refer to the same number, or the same portion of the pie, but have different denominators. Higher terms are used for adding and subtracting fractions.

- Step 1: Find the LCM of all denominators.
- Step 2: Divide the LCM by the original denominator to find the multiplier.
- Step 3: Multiply both the numerator and denominator by the multiplier.

Example 17: Write $\frac{2}{9}$, $\frac{2}{3}$, $\frac{5}{12}$ as equivalent fractions with a common denominator.

$$9 = 3^2$$

$$12 = 2^2 \cdot 3$$

$$1.CM = 3^2 \cdot 2^2 = 36$$

Step 2: Divide LCM by the denominators to get the multipliers:

36/9 = 4 for the first fraction

36/3 = 12 for the second fraction

36 / 12 = 3 for the third fraction

Step 3: First fraction:
$$\frac{2}{9} \cdot \frac{4}{4} = \frac{8}{36}$$
; Second fraction: $\frac{2}{3} \cdot \frac{12}{12} = \frac{24}{36}$; Third fraction: $\frac{5}{12} \cdot \frac{3}{3} = \frac{15}{36}$

Answer:
$$\frac{8}{36}, \frac{24}{36}, \frac{15}{36}$$

9. Write equivalent fractions (lower terms) = Reduce to lowest terms.

Equivalent fractions refer to the same number, or the same portion of the pie, but have different denominators. Final answers should always be reduced to lowest terms.

- Step 1: Find a common factor of the numerator and denominator.
- Step 2: Divide both by the common factor.
- Step 3: Repeat until there are no common factors except 1.

Note: To be efficient, divide by the largest common factor possible.

Example 18: Reduce
$$\frac{24}{42}$$

Quickest way:

Step 2: 24 / 6 = 4 and 42 / 6 = 7, giving
$$\frac{4}{7}$$

Answer:
$$\frac{4}{7}$$

Slower way:

Step 2:
$$24/2 = 12$$
 and $42/2 = 21$, giving $\frac{12}{21}$

Step 3:
$$12/3 = 4$$
 and $21/3 = 7$, giving $\frac{4}{7}$

Answer:
$$\frac{4}{7}$$

10. Round decimals

Caution: Only round when instructions say to round.

Step 1: Identify the number in the place to which the number is to be rounded.

Step 2: Check by looking one digit to the right.

Step 3: If the check digit is 5 or more, add one to the digit to which it is rounded.

If the check digit is 4 or fewer, leave the digit to which it is rounded the same.

Step 4: Remove check digit and all digits to the right.

Example 19: Round 1.69843 to the nearest thousandth.

Thousandth is 8.

Check digit is 4, leave 8 unchanged.

Answer: 1.698

Example 20: Round 1.69843 to the nearest hundredth.

Hundredth is 9.

Check digit is 8, add one to 9.

Answer: 1.70

11. Convert a fraction to a decimal

To convert a fraction to a decimal, divide the numerator by the denominator. Use repeating bars when needed.

Example 21: Convert
$$\frac{3}{8}$$
 to a decimal.

$$3/8 = 0.375$$

Answer: 0.375

Example 22: Convert $\frac{7}{9}$ to a decimal.

$$7/9 = .77777...$$

Answer: $0.\overline{7}$

12. Convert a decimal to a fraction

Step 1: Identify the name of the right-most decimal place.

Step 2: Use that name to find the denominator.

Step 3: Remove the decimal point and use the number as the numerator.

Step 4: Write fraction and reduce to lowest terms.

Example 23: Write 0.13 as a fraction.

3 is in the hundredths place. Use 100 as the denominator.

.13 without a decimal point is 13. Use 13 as the numerator.

$$\frac{13}{100}$$
 does not reduce.

Answer: $\frac{13}{100}$

Example 24: Write 0.168 as a fraction.

8 is in the thousandths place. Use 1000 as the denominator.

.168 without a decimal point is 168. Use 168 as the numerator.

Write and reduce:
$$\frac{168}{1000} = \frac{42}{250} = \frac{21}{125}$$

Answer: $\frac{21}{125}$

13. Convert a percent to a decimal

Move decimal point two places to the left.

Example 25: Write 95% as a decimal. Answer: 0.95

Example 26: Write 5.2% as a decimal. Answer: 0.052

Example 27: Write 135% as a decimal. Answer: 1.35

Example 28: Write 1.02% as a decimal. Answer: 0.0102

14. Convert a decimal to a percent

Move decimal point two places to the right.

Example 29: Write 0.63 as a percent. Answer: 63%

Example 30: Write 0.006 as a percent. Answer: 0.6%

Example 31: Write 72 as a percent. Answer: 7200%

Example 32: Write 1.03 as a percent. Answer: 103%

15. Convert percent to fraction

Step 1: Write as decimal.

Step 2: Write decimal as fraction.

Step 3: Reduce fraction.

Example 33: Write 95% as a fraction.

Decimal 0.95
Fraction
$$\frac{95}{100} = \frac{19}{20}$$

Answer: $\frac{19}{20}$

Example 34: Write 5.2% as a fraction.

Decimal: 0.052

Fraction:
$$\frac{52}{1000} = \frac{26}{500} = \frac{13}{250}$$
 Answer: $\frac{13}{250}$

Example 35: Write 135% as a fraction.

Decimal: 1.35

Fraction:
$$\frac{135}{100} = \frac{27}{20}$$

Answer: $\frac{27}{20}$

Example 36: Write 1.02% as a fraction.

Decimal: 0.0102

Fraction:
$$\frac{102}{10000} = \frac{51}{5000}$$

Answer:
$$\frac{51}{5000}$$

16. Convert fraction to percent

Method 1: Convert fraction to decimal, then decimal to percent.

Example 37: Write $\frac{3}{4}$ as percent.

Decimal: 3/4 = .75

Percent: 75%

Answer: 75%

Example 38: Write $\frac{2}{9}$ as percent.

Decimal: $2/9 = .\overline{2}$

Percent: $22.\overline{2}\%$ or $22\frac{2}{9}\%$

Answer: $22.\overline{2}\%$ or $22\frac{2}{9}\%$

Method 2 (only if denominator is a factor of 100): Find multiplier to make denominator 100.

Example 39: Write $\frac{3}{4}$ as percent.

Find multiplier: 100/4 = 25

Multiply numerator and denominator: $\frac{3 \cdot 25}{4 \cdot 25} = \frac{75}{100}$

Numerator is percent ("per one hundred").

Answer: 75%

Sets of Numbers

Natural Numbers = $\{21, 2, 3, ...\}$ = IN Whole Numbers = $\{20, 1, 2, 3, ...\}$ = W

The phrase "natural numbers" has no letter 0, just as the set has no 0.

The phrase "whole numbers" has a letter 0, just as the set has the number 0.

Integers = $\frac{1}{2}$... $\frac{3}{2}$, $\frac{1}{2}$, $\frac{3}{2}$... $\frac{3}{2}$ = $\frac{1}{2}$ then German for counting, "Zahlen".

Rational Numbers: can be written as a fraction of decimal either terminates or repeats.

Examples: 1= + is rational

2=2 is rational

Note 1 is undefined 0 = 0 is rational

== .5 is rational

 $\frac{3}{4}$ = .75 is rational

.12306 = 12306 is rational

 $\overline{3} = \frac{1}{3}$ is rational

 $_{0}\overline{6} = \frac{2}{3}$ is rational

T = 1 is rational

 $\overline{2} = \frac{2}{9}$ is rational

.4 = 4 is rational

 $.5 = \frac{5}{9}$ is rational

 $\overline{7} = \frac{7}{9}$ is rational

= Q

rational =>
root ratio =>
fraction =>
division =>
quotient =>
Q

All of
these are
rational
because
they can
be written
ous a
fraction
(Afen if
not in
lowest
terms)

= 8 = 8 is rational

 $37 = \frac{37}{99}$ is rational

. 207 = even if you don't know what the fraction is, repeating decimal means it can be written as a fraction.

Irrational Numbers: cannot be written as a fraction

nor repeats. A

examples: Tr

12

Real Numbers = TR = all rationals and all irrationals together.

Math 45 Handout for 1.3 The Number Systems and the Real Number Line

- 1. Classify numbers as natural, whole, integer, rational, irrational, real
- 2. Plot points with fractional or decimal parts on a number line.
- 3. Use inequalities to compare two numbers.
- Evaluate expressions with absolute values.

Write the set.

1) C is the set of integers between -5 and 3. $\{-4, -3, -2, -1, 0, 1, 2\}$ the elements of B that include endpoints!

List all the elements of B that belong to the given set.

2)
$$B = \{11, \sqrt{7}, -24, ... \frac{0}{7}, \sqrt{4}, 0.33, -7\pi, 0.444....\}$$
 $\frac{0}{7} = 0$ $\sqrt{4} = 2$.444... = $\frac{4}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ Natural Numbers 11, 0, 2- $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ Integers 11, 0, 2- $\frac{1}{9}$ $\frac{1}{9}$ Integers 11, -24, 0, 2- $\frac{1}{9}$

Answer True or False to the statement.

- 3) Every rational number is an integer. false ex: 2
- 4) Every whole number is a real number. + rue
- 5) Some rational numbers are irrational. +alse
- 6) Some rational numbers are integers.

Plot the points in the set on a real number line. Use correct scale and label each point.

$$7\left\{-1,\frac{5}{3},-\frac{17}{3},-3\right\} = \left\{-1, \left[\frac{2}{3}, -5\frac{7}{3}, -3\right\} \Rightarrow -5\frac{2}{3} < -3 < -1 < \left[\frac{2}{3}\right]$$
 step 1: Simplify

$$-6\frac{-17}{3}$$
 -5 -4 -3 -2 -1 0 1 $\frac{5}{3}$ 2 3

8)
$$\{-1.75, 5.25, 1.3\} = \{-1\frac{3}{4}, 5\frac{1}{4}, 1, 3\} \Rightarrow -1\frac{3}{4} < 1 < 3 < 5\frac{1}{4}$$
 and mark evenly spaced integers covering all numbers in list.

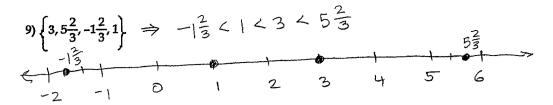
5.25

Step 4: Plot dots on the line

step 1: Simplify all numbers · Stepa: Write numbers in order from smallest to largest and mark evenly spaced integers covering all numbers in list. Step4. Plot dots on the line

Step5: To plot numbers with fraction parts, identify the two integers on either side e.g. 62-53<-5

Use denominator to divide the segment. step 6: Label all points:



Replace the ? with the correct symbol >, <, =.

10) -7 ? 0
$$\angle$$

$$13)\frac{8}{2}$$
? $\frac{12}{3}$ $+ = 4$

$$\frac{14)^{\frac{2}{5}}?^{\frac{5}{9}}}{45} \qquad \frac{18}{45} < \frac{25}{45}$$

$$15) - \frac{5}{7}? - \frac{3}{10} \quad \frac{-50}{70} \quad < \quad \frac{-21}{70}$$

$$16) - \frac{3}{7}? - \frac{5}{8} \qquad -\frac{24}{56} \qquad > -\frac{35}{56}$$

19) |-11| ?
$$\frac{33}{-3}$$
 | | > -||

Math 45: Useful Math Terminology

Operation: add, subtract, multiply, divide, exponent, radical, absolute value, etc.

Order of Operations: Rules for which part of a calculation to do first, and then what order to proceed

Variable: A letter used to represent any or a specific number, often unknown

Expression: A combination of variables and numbers with no equal sign, though operations are permitted

Substitute: Replace a variable or an expression by a variable or expression which is known to be equal

Evaluate: Give a number answer as a result, often by substituting given values for a variable then performing any operations

Terms: Parts of an expression that are separated by addition (rewrite subtraction as addition)

Distribute: Multiply a single term by all terms within parentheses that follow or precede it

Factors: Parts of a term that are separated by multiplication (rewrite division as multiplication)

Factor: Find the prime factors (numbers or polynomials) which multiply to give the original expression

Reduce: Put a fraction in lowest terms. A reduced fraction has no common factors, decimals or fractions within fractions

Constant: An expression which is a number without variables

Coefficient: A number, especially if multiplied times a variable or product of variables

Equation: Two expressions separated by an equal sign

Inequality: (a) One of the four symbols <, >, \le , \ge

(b) Two expressions separated by an inequality symbol

Solve: Find all solutions, meaning the values for a variable which make an equation or inequality true. Solution may be a list, set, interval, or a graph.

Isolate: Perform operations to both sides of an equation or inequaltiy so the isolated item is alone on one side

Set equal to zero: Write an equation by writing a given expression, an equal sign, and a zero

Rational: A number or expression which is or can be written as a fraction

Simplify: Final form for an answer. Reduce fractions, distribute, combine like terms, evaluate operations. If simplifying a rational expression, factor and divide out common factors, then leave final answer factored.

Interval: A set of all values between two given endpoints. Notation will specify if endpoints are included.

Ordered Pair or Point: A pair of numbers in parentheses, separated by a comma, denoting the x- and y-coordinates on a rectangular coordinate graph